The Wandering Exponent of a One-Dimensional Directed Polymer in a Random Potential with Finite Correlation Radius

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Abstract

We consider a one-dimensional directed polymer in a random potential which is characterized by the Gaussian statistics with the finite size local correlations. It is shown that the well-known Kardar's solution obtained originally for a directed polymer with δ-correlated random potential can be applied for the description of the present system only in the high-temperature limit. For the low temperature limit we have obtained the new solution which is described by the one-step replica symmetry breaking. For the mean square deviation of the directed polymer of the linear size L it provides the usual scaling $\overline{x^2} \sim aL^{2\zeta}$ with the wandering exponent $\zeta = 2/3$ and the temperature-independent prefactor.

1 Introduction

In a wide variety of physical systems one is interested in the behaviour of a fluctuating linear object (with finite line tension) interacting with a quenched random potential. The object under consideration may be a dislocation in a crystal, a domain wall in a two-dimensional magnet, a vortex line in a superconductor, a fluxon line in an extended Josephson junction and so on, but following Ref. [1] this class of problems is traditionally discussed in terms of a directed polymer in random media or symply "directed polymer".

Quite naturally the best understanding has been achieved for the simplest one-dimensional case when the displacements of a directed polymer can occur only in one direction. In such case a directed polymer in continuous approximation can be described by the Hamiltonian

$$H[x(t), v] = \int_0^L dt \left\{ \frac{J}{2} \left(\frac{dx}{dt} \right)^2 + v[x(t), t] \right\} \tag{1.1}$$

where J is the linear tension, t is the longitudal coordinate ($0 \leq t \leq L$) and $x(t)$ is the transverse displacement of a polymer with respect to a straight line. The simplest (or maybe one should better say the most easily treatable) assumption on the distribution of random potential $v(x, t)$ consists in taking it to be Gaussian with

$$\overline{v(x, t)} = 0; \qquad \overline{v(x, t) v(x', t')} = 2V(x - x')\delta(t - t'). \tag{1.2}$$

Here and further on an overbar denotes the average over the realizations of quenched random potential.

Let us assume that at $t = 0$ the position of a polymer is fixed: $x(0) \equiv 0$. Then the quantity of interest is the typical deviation of the polymer "trajectory" $x(t)$ from the origine. More precisely, one would like to know the dependence on L of the average square deviation of the polymer at the ending point $x = L$, which in the limit $L \to \infty$ is expected to be described by the simple scaling:

$$\overline{\langle x^2(L) \rangle} \sim aL^{2\zeta} \tag{1.3}$$

where angular brackets denote the average over thermal fluctuations, and ζ is the so-called wandering exponent.

In the absence of the random potential the situation is trivial and the wandering exponent ζ is equal to $1/2$. In this case the trajectory deviates from the origine only due to the thermal fluctuations, the prefactor a in the scaling law (1.3) is proportional to the temperature, so at zero temperature $\langle x^2(L) \rangle = 0$.

In the presence of a quenched random potential the situation is getting much more complicated. Now besides the thermal fluctuations, the trajectory is pushed away from

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the origine also due to the randomness in the background potential landscape, so that the scaling law (1.3) could be governed by a new non-trivial wandering exponent. Moreover, since for a generic random potential the ground state trajectory of the Hamiltonian (1.1) typically drifts away from the origine, the scaling law (1.3) can be expected to hold also in the zero temperature limit.

It is widely believed that for a whole class of *locally correlated* random potentials (such that the function $V(x)$ in (1.2) is quickly decaying for $|x| \to \infty$) the wandering exponent ζ is universal and equal to 2/3. This conclusion is based on the finite temperature exact results of Refs. [2, 3] and has been also confirmed by zero temperature numerical simulations of the discrete version of the directed polymer problem [1] (see also Ref. [4] for later references). However, both the calculation based on the reduction to damped Burgers' equation with conservative random force [2] and Bethe ansatz calculation in terms of replica representation [3] are valid only for the case of stricktly δ-functional correlations of random potential.

In this work we present an attempt to generalize the second of these solutions to a more physical situation when correlations of random potential have finite correlation radius (in transverse direction).

In Sec. 2 we review some details of the approach developed by Kardar in Ref. [3] for δ-functional correlations of random potential and use it to find the temperature dependence of a prefactor in Eq. (1.3) which turns out to be of the form

$$a \propto T^{-2/3} \tag{1.4}$$

In Sec. 3 the applicability of Kardar's solution [3] for the approximate description of the system with the finite radius r of the random potential correlations is discussed. We demonstrate that it can be used only in the high temperature limit $T \gg T_0 \propto r^{2/3}$, whereas at low temperatures the solution has to have essentially different structure which in principle could lead to the change in wandering exponent. The impossibility to apply the description with the help of the Kardar's solution at arbitarily low temperatures follows already from Eq. (1.4). One can expect that if in the low temperature limit the typical trajectory goes away from the origine its drift should be determined by the quenched random potential and not by the effects of the thermal fluctuations. It means that in the system with reasonable short-scale *regularization* the divergence of the prefactor suggested by Eq. (1.4) at low temperatures can be expected to saturate.

In Sec. 4 the low temperature solution of the regularized problem is found for the particular choice of the random potential correlation function $V(x)$. The form of this solution can be described in terms of the effective one-step replica symmetry breaking ansatz. In this case one recovers the scaling law (1.3) with the same wandering exponent $\zeta = 2/3$ but with a *temperature-independent* prefactor.

2 The solution of the unregularized problem

2.1 The relation between exponents

The idea of the Kardar's approach [3] is based on the indirect calculation of the wandering exponent ζ by analyzing the scaling of the typical sample to sample *fluctuations* of the free energy. Suppose that the typical fluctuations of the free energy (produced by the random potential) scale as

$$\delta F \propto L^{\omega} \tag{2.1}$$

where the exponent ω is known. On the other hand, if the typical deviation of the trajectory from the origine is equal to x, then the loss of the energy due to the elastic term in the Hamiltonian (1.1) must be of the order of Jx^2/L. Ballancing the two energies, one can write the following estimate:

$$\overline{\langle x^2 \rangle} \sim \frac{L \, \delta F}{J} \propto L^{\omega+1} \tag{2.2}$$

Then, according to the definition of the wandering exponent (1.3), one finds the following simple relation between the two exponents:

$$2\zeta = \omega + 1 \tag{2.3}$$

2.2 The replica method

The scaling of the free energy fluctuations with the size of the system L can be relatively easy investigated in terms of the replica method [5, 6]. To this end one has to calculate the average

$$Z(n) \equiv \overline{Z^n[v]} \tag{2.4}$$

of the n-th power of the partition function:

$$Z[v] = \int_{0<t<L} Dx(t) \exp\left\{ -\frac{H[x(t), v]}{T} \right\} \tag{2.5}$$

obtained by the integration over all the trajectories with $x(0) = 0$.

According to the definition of the free energy:

$$F = -T \ln Z[v] \tag{2.6}$$

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the replica partition function $Z(n)$ can be represented as follows:

$$Z(n) = \overline{\exp\left[-\frac{n}{T}F\right]} \qquad (2.7)$$

The average in Eq. (2.7) as everywhere above is calculated over the realizations of random potential $v(x,t)$. On the other hand the free energy $F \equiv F[v]$ is itself the sample-dependent random quantity, whose distribution function we shall denote as $P(F)$. Then Eq. (2.7) can be rewritten as

$$Z(n) = \int dF P(F) \exp\left[-\frac{n}{T}F\right] \qquad (2.8)$$

which is nothing else but the Laplace transform of the free energy distribution function $P(F)$.

The free energy corresponding to the replica partition function (2.4) can be naturally defined as:

$$F(n) = -T \ln Z(n) \qquad (2.9)$$

Although this quantity can be calculated only for integer n, according to the standart ideology of the replica approach it has to be considered as a function of the *continuous* parameter n which implies a necessity of an analytic continuation in n.

Let us represent the free energy $F(n)$ of the replicated system as a series in powers of the replica parameter n:

$$F(n) = \sum_{k=1}^{\infty} \frac{F_k}{k!} n^k \qquad (2.10)$$

Then, taking the k-th derivative over n at $n = 0$ from both sides of the Eq.(2.9) for the k-th order of the free energy fluctuations one finds:

$$F_k = T^{1-k} \overline{\overline{F^k}} \qquad (2.11)$$

where double overbar denotes the irreduceble average.

2.3 The Bethe ansatz type solution

In the framework of replica approach the statistical meachanics of the *irregular* system is analyzed by considering the statistical mechanics of the *regular* system in which the disorder manifests itself in the form of the interaction between n identical replicas of the original system. For the system described by the Hamiltonian (1.1) and the Gaussian statistics of the random potential, Eq.(1.2), the averaging of $Z^n[v]$ over disorder

leads to the expression for $Z(n)$ the form of which corresponds to the following replica Hamiltonian:

$$H_{repl} = \int_0^L dt \left\{ \frac{J}{2T} \sum_{a=1}^n \left(\frac{dx_a}{dt} \right)^2 - \frac{1}{T^2} \sum_{a,b=1}^n V[x_a(t) - x_b(t)] \right\}. \tag{2.12}$$

Eq. (2.12) has a form of the Eulidean (imaginary time) action describing the quantum-mechanical system of n particles with the mass J/T and interaction $V(x)/T^2$. The same system can be described by the quantum-mechanical (operator) Hamiltonian

$$\hat{H} = -\frac{T}{2J} \sum_{a=1}^n \nabla_a^2 - \frac{1}{T^2} \sum_{a,b=1}^n V(x_a - x_b) \tag{2.13}$$

which for the classical partition function defined by the Hamiltonion (2.12) plays the role of the transfer matrix.

In the limit of infinite size ($L \to \infty$) the free energy of a system (for any boundary conditions) is dominated by the highest eigenvalue of transfer matrix or, in our case, by the lowest eigenvalue E_0 of the quantum-mechanical Hamiltonian (2.13)

$$F(n) = TE_0(n)L \tag{2.14}$$

For any integer n the lowest eigen-value corresponds to the fully symmetric (nodeless) wave-function which for the case of local correlations in x direction

$$V(x) = u\delta(x) \tag{2.15}$$

has been found exactly by Kardar [3]:

$$\Psi_0[x_a] = \exp \left(-\text{æ} \sum_{a,b=1}^n |x_a - x_b| \right) \tag{2.16}$$

where for our choice of notation

$$\text{æ} = \frac{Ju}{T^3} \tag{2.17}$$

The energy of this state is equal to

$$E_0(n) = -\frac{V(0)}{T^2}n - \frac{Ju^2}{6T^5}n(n^2 - 1) \tag{2.18}$$

where the first term describes the trivial contribution to $E(n)$ related to the terms with $a = b$ in the second sum in Hamiltonian (2.13).

Substitution of Eq. (2.18) into Eq. (2.14) gives

$$F(n) = F_1 n + \frac{1}{6} F_3 n^3 \tag{2.19}$$

where

$$F_1 = \left[-\frac{V(0)}{T} + \frac{Ju^2}{6T^4} \right] L \tag{2.20}$$

and

$$F_3 = -\frac{Ju^2}{T^4} L \tag{2.21}$$

Comparison of Eq. (2.11) with Eq. (2.19) shows that for $L \to \infty$ the average free energy (per unit length) of a random polymer

$$f \equiv \lim_{L \to \infty} \frac{\overline{F(L)}}{L} \tag{2.22}$$

is given by the linear in n contribution to $F(n)$:

$$f = \frac{1}{L} \lim_{n \to 0} \frac{F(n)}{n} = \frac{F_1}{L} = -\frac{V(0)}{T} + \frac{Ju^2}{6T^4} \tag{2.23}$$

in which the first (formally divergent) term always dominates. Therefore the average free energy of the system could be defined only after proper short-scale regularization of the starting Hamiltonian.

However, the fluctuations of the free energy are quite well defined without any regularization. According to Eqs. (2.11) and (2.21) the typical value of the free energy fluctuations can be estimated as:

$$\delta F \sim \left(\left| \overline{\overline{F^3}} \right| \right)^{1/3} = \left(T^2 |F_3| \right)^{1/3} = \left(\frac{Ju^2}{T^2} \right)^{1/3} L^{1/3} \tag{2.24}$$

Therefore, according to Eq. (2.2) for the average square deviation of the trajectory one finds the following result:

$$\overline{\langle x^2 \rangle} \sim \left(\frac{u}{JT} \right)^{2/3} L^{4/3} \tag{2.25}$$

3 Introduction of the regularization

Apparently the divergence of the average free energy, Eq.(2.23), is removed if one takes into account that in a physical system correlations of random potential should be described by a smooth function with a finite correlation radius [and therefore a finite value of $V(0)$]. However in such case the quantum-mechanical problem defined by Eq. (2.13) cannot be solved exactly. Nonetheless it seems reasonable to assume that for narrow enough $V(x)$ one can still use the expression (2.23) in which now u should stand for

$$u = \int_{-\infty}^{+\infty} dx \, V(x) \tag{3.1}$$

It is easy to understand that such approximate description [based on Eqs. (2.16)-(2.21)] at low temperatures has to fail. One has to remember that in case of the δ-functional interaction the wave function (2.16) is constructed as a generalization of two-particle problem wavefunction

$$\Psi(x_1, x_2) = \exp(-\ae|x_1 - x_2|) \tag{3.2}$$

On the other hand in the case of a rectangular well:

$$V(x) = \begin{cases} V & \text{for } |x| < r \\ 0 & \text{for } |x| > r \end{cases} \tag{3.3}$$

(for which $u = 2Vr$) the wave function of the two-particle problem for $|x_1 - x_2| > r$ also has the form (3.2) with

$$\ae = \frac{1}{r} g\left(\frac{2T_0^3}{T^3}\right) \tag{3.4}$$

where

$$g(z) \approx \begin{cases} z^{1/2} & \text{for } z \ll 1 \\ z & \text{for } z \gg 1 \end{cases} \tag{3.5}$$

and

$$T_0 = (JVr^2)^{1/3} \tag{3.6}$$

It is not hard to check that the condition $\ae r \ll 1$ for this wave-function to be wide in comparison with the well width coinsides with the condition $T \gg T_0$. In such case the value of $\ae$ given by Eqs. (3.4)-(3.6) coinsides with (2.17). In the opposite limit $T \ll T_0$ ($\ae r \gg 1$) the two-particvle wave-function is almost completely localized inside the well

and cannot be used as a building block for the construction of the solution of n-particle problem.

This gives a clear indication that for *arbitrary* finite-width form of the function $V(x)$ describing the correlations of random potential the applicaton of the Kardar's solution for the description of random polymer can work only at high enough temperatures whereas in the low temperature limit the solution should be different.

Quite paradoxically the differentiation of Eq. (2.23) shows that for $T \gg T_0$ the free energy defined by Eq. (2.23) corresponds to negative enthropy. One should not be too scared of that property since in the approach discussed above the free energy of a directed polymer is calibrated in such a way that in the absence of the disorder it is equal to zero. Therefore the total free energy will be given by Eq. (2.23) *plus* the free energy in absence of disorder. This second term will give the positive contribution to the enthropy which will overcome the negative contribution from Eq. (2.23).

4   The low temperature solution of the regularized problem

Let us now consider the n-particle problem defined by Hamiltonian (2.13) where

$$V(x) = V \left[1 - b \frac{x^2}{r^2} \right] \tag{4.1}$$

for $|x| < r$ and is equal to zero elsewhere. Here in comparison with Eq. (3.3) we have introcuded a finite curvature of the potential inside the well which is described by an additional free parameter b ($0 < b \leq 1$). At some stage of calculation b will be assumed to be much smaller than one. For $V(x)$ of the form (4.1)

$$u = 2 \left(1 - \frac{b}{3} \right) V r \sim 2 V r \tag{4.2}$$

and therefore the characteristic temperature T_0 defining the range of applicability of Kardar's approach ($T \gg T_0$) still can be chosen in the form (3.6).

The characteristic frequency for small oscillations at the bottom of such truncated parabolic well is given by

$$\Omega = \sqrt{\frac{2bV}{JTr^2}} \tag{4.3}$$

and increases with decrease of T much slower than the depth of the well $W = V/T^2$. Thus the limit of low temperatures may correspond to the case when all particles are localized near the bottom of the well. In such limit the ground state energy $E(n)$ for

the n-particle system can be rather accurately found by assuming that Eq. (4.1) holds for all $x_a - x_b$.

In such aproximation the ground state wave-function has a form

$$\Psi[x_a] = \exp\left[-\frac{1}{A\sqrt{2n}}\sum_{a,b=1}^{n}(x_a - x_b)^2\right]; \quad A = \frac{\Omega}{W}\frac{r^2}{b} \tag{4.4}$$

whereas its energy is given by

$$E(n) = -Wn^2 + \Omega\sqrt{\frac{n}{2}}(n-1) \tag{4.5}$$

(cf. with Refs. [7] and [8]). In the following it will be convenient to keep in mind that the ratio of Ω and W can be expressed as

$$\frac{\Omega}{W} = \left(2b\frac{T^3}{T_0^3}\right)^{1/2} \tag{4.6}$$

It is not hard to find by a straightforward calculation that for $\Psi(x)$ of the form (4.4)

$$\langle(x_a - x_b)^2\rangle = \frac{A}{\sqrt{2n}} = \frac{1}{\sqrt{2n}}\frac{\Omega}{W}\frac{r^2}{b} \tag{4.7}$$

so for $\Omega/W \ll b$ (that is $T \ll b^{1/3}T_0$) $\Psi(x)$ is indeed nicely localized at the bottom of the well for any integer n and all corrections to $E(n)$ due to non-parabolicity can be only exponentially small.

On the other hand one can easily see why the limit $n \to 0$ is dangerous. The width (4.7) of the wavefunction (4.4) grows with decrease in n and becomes comparable with the width of the well $2r$ at $n \sim b^{-1}(T/T_0)^3$ and therefore for smaller n the ground state wave-function should have essentially different form. The simplest way to let the particles enjoy their mutual attraction while keeping their number in the well not too small consists in splittting them into n/k infinitely separated blocks of k particles. The energy $E(n,k)$ of such state with broken replica symmetry is given by

$$E(n,k) = \frac{n}{k}E(k) = n\left[-Wk - \Omega\frac{1-k}{\sqrt{2k}}\right] \tag{4.8}$$

and has extremum (maximum) as a function of k.

Variation of the polymer free energy per unit length:

$$f(k) = T\left[-Wk - \Omega\frac{1-k}{\sqrt{2k}}\right] \tag{4.9}$$

with respect to k gives an equation for the position of the maximum:

$$-W + \Omega\frac{1+k}{(2k)^{3/2}} = 0 \tag{4.10}$$

the solution of which for $\Omega/W \ll 1$ has a form

$$k_* \approx \frac{1}{2}\left(\frac{\Omega}{W}\right)^{2/3} = \frac{1}{2}(2b)^{1/3}\frac{T}{T_0} \tag{4.11}$$

Substitution of Eq. (4.11) into Eq. (4.7) then shows that for

$$T \ll b^{2/3}T_0 \tag{4.12}$$

the replicas belonging to the same block are indeed tightly bound to each other:

$$\langle(x_a - x_b)^2\rangle \approx \left(\frac{\Omega}{W}\right)^{2/3}\frac{r^2}{b} \approx \frac{T}{b^{2/3}T_0}r^2 \ll r^2 \tag{4.13}$$

so the whole picture is really self-consistent.

Substitution of Eq. (4.11) into Eq. (4.9) gives the temperature independent expression:

$$f \approx -\frac{3}{2}(2b)^{1/3}\frac{V}{T_0} \tag{4.14}$$

which shows that in order to find the temperature dependence of f in low temperature limit we have to solve Eq. (4.10) more accurately. That gives

$$k_* \approx \frac{1}{2}\left(\frac{\Omega}{W}\right)^{2/3} + \frac{1}{4}\left(\frac{\Omega}{W}\right)^{4/3} \tag{4.15}$$

and

$$f \approx -\frac{3}{2}(2b)^{1/3}\frac{V}{T_0} + \frac{1}{4}(2b)^{2/3}\frac{V}{T_0^2}T \tag{4.16}$$

The idea to consider the state in which n replicas are split into n/k infinitely separated blocks of k particles has been introduced by Parisi [9], who however applied it only to the case of local interaction (2.15) (δ-functional correlations in terms of original problem) and discovered that free energy as a function of k has extremum at $k = 0$. That is equivalent to considering all replicas belonging to the same block right from the beginning. Our analysis shows that smearing of the interaction potential leads (at low enough temperatures) to the shift of the extremum to non-trivial value of k ($0 < k < 1$) corresponding to splitting of replicas into blocks (that is to replica symmetry breaking).

Although we have found that in the exremal solution the particles split into k separate blocks there are no reasons for these blocks to be infinitely separated from each other. The presence of strong attraction between the particles in each block makes it possible to

consider such block as a complex particle with the mass kJ/T, the interaction between these complex particles being given by $k^2V(x)/T^2$. The last expression can be expected to be very accurate when we consider the temperature interval (4.12) in which the distances between the particles inside each block are much smaller than the well radius r.

Therefore at low temperatures the behaviour of our system in which the particles are assumed to be tightly bound in n/k separate blocks can be described by the Hamiltonian (2.13) in which

$$J \to kJ; \quad V(x) \to k^2V(x); \quad u \to k^2u \tag{4.17}$$

Our earlier experience tells us that for some values of parameters such system can be rather accurately described by the wave function of the form (2.16) in which now x_a stand for coordinates of different blocks. The energy of such state will be given by Eq. (2.18) in which substitutions (4.17) and $n \to n/k$ have to be made with the first term being substituted by Eq. (4.8):

$$E(n,k) = n \left\{ -Wk - \Omega\frac{1-k}{\sqrt{2k}} - 2B\frac{W^3}{\Omega^2}k^4 \left[\left(\frac{n}{k}\right)^2 - 1 \right] \right\} \tag{4.18}$$

where

$$B = \frac{2b}{3}\left(1 - \frac{b}{3}\right)^2 \tag{4.19}$$

is a small parameter if b is small.

All this leads to appearence in the expression for $f(k)$ of one more term [in comparison with Eq. (4.9)]:

$$f(k) = T\left[-Wk - \Omega\frac{1-k}{\sqrt{2k}} + 2B\frac{W^3}{\Omega^2}k^4 \right] \tag{4.20}$$

which describes the contribution related to mutual interaction between the blocks. Substitution of Eq. (4.11) into the saddle-point equation

$$-W + \Omega\frac{1+k}{(2k)^{3/2}} + 8B\frac{W^3}{\Omega^2}k^3 = 0 \tag{4.21}$$

which is obtained by variation of Eq. (4.20) shows that for $B \ll 1$ (that is for $b \ll 1$) the maximum of $f(k)$ still exists and in the lowest order in b the position of this maximum is still given by Eq. (4.11).

The applicability of such approach requires that the distances between the blocks should be much larger than the size of the well (exactly in same way as when Kardar type solution is constructed from the separate particles and not from the blocks):

$$\text{æ}r \ll 1 \tag{4.22}$$

Sustitution of Eqs. (2.17), (4.17) and (4.11) into Eq. (4.22) then reduces it to condition

$$\frac{b}{2}\left(1 - \frac{b}{3}\right) \ll 1 \tag{4.23}$$

which apparently is equivalent to the same condition $b \ll 1$.

Strictly speaking the expression for $f(k)$ given by Eq. (4.20) has also another extremum (minimum) at

$$k = k_{**} \approx \frac{1}{B^{1/3}}k_* \tag{4.24}$$

but analogous analysis shows that

$$\mathit{æ}(k_{**})r \approx \frac{3}{4}\left(1 - \frac{b}{3}\right)^{-1} \sim 1 \tag{4.25}$$

and therfore this second extremum takes place in the domain of parameters where expression (4.20) based on the assumption (4.22) can no longer be trusted.

The form of Eq. (4.18) shows that in the considered case the only non-linear (in n) contribution to $E(n)$ is also of the third order in n and corresponds to

$$F_3 = -\frac{4VT_0^3}{T^4}k^2 L \tag{4.26}$$

where we assume $b \ll 1$. Using the saddle-point value of the parameter k one gets:

$$F_3(k_*) = -(2b)^{2/3}\frac{VT_0}{T^2}L \tag{4.27}$$

Correspondingly, for the typical value of the free energy fluctuations one obtaines the following temperature independent result:

$$\delta F \sim \left(T^2|F_3|\right)^{1/3}L^{1/3} = C_0 L^{1/3} \tag{4.28}$$

where

$$C_0 = (2b)^{2/9}(VT_0)^{1/3} \tag{4.29}$$

Finally, for the mean square deviation of the polymer trajectory one again finds $\overline{\langle x^2\rangle} \sim aL^{4/3}$ with the temperature independent prefactor

$$a = \frac{C_0}{J} = \left(2b\frac{V^2 r}{J^4}\right)^{2/9} \tag{4.30}$$

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5 Conclusion

Thus we have demonstrated that in the case when random potential correlations are characterized by finite correlation radius r the solution at low temperatures has essentially different structure than at high temperatures. Nonetheless the value of the wandering exponent in both cases is the same: $\zeta = 2/3$. In contrast to the high temperature limit for which the prefactor a in scaling law (1.3) is temperature dependent: $a \propto T^{-2/3}$, in the low temperature limit it saturates at finite value $a \propto r^{2/9}$.

Since the value of the wandering exponent for both regimes is the same there are no reasons for the sharp transition between these regimes. At very low temperatures the solution is characterized by the one-step replica symmetry breaking that is the replicas are splitted into k well separated blocks. With growth of temperature the distance between the blocks becomes comparable with the size of each block. At higher temperatures the replica symmetry breaking phenomena can manifest itself only in slight modulation of the distance between nearest replicas in comparison with what follows from the "replica symmetric" wave-function (2.16).

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